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Interdisciplinary collaboration between engineering, mathematics and science

SEMS Research Highlights

Functional Equations Defined on Abstract Structures

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This newsletter presents the research conducted within the School of Engineering, Mathematics and Science (SEMS) at Robert Morris University (RMU). It covers various relevant topics including: interdisciplinary efforts, successful research grants, student research, posters and papers, journal publications, presentations at national and international conferences, contribution to professional societies, STEM educational research, industrial consulting collaborations and applied research.

All mathematicians, scientists, and engineers have at some point in their lifetime built a model. Most models are built by selecting or creating at least one equation that aims to reproduce (to some degree) a reality. Models often have multiple conditions that must be met. Because of this they exhibit inconsistencies or technical difficulties that make the situation appear arbitrary. Functional equations are a mathematical tool used to help prevent these inconsistencies and form more rigorous models. They seek to describe complex mathematical relationships in simple terms. Functional equations arise in areas such as computer science, economics, statistics, information theory, probability theory, signal processing, and pattern recognition. Solving a functional equation is not like solving a standard mathematical equation. Rather than solving for the value of a variable, one tries to find the types or classes of functions that satisfy the given equation with respect to certain conditions. An example can be seen above. Because of their vast spectrum of applications, it is common for mathematicians to study functional equations defined on different mathematical structures. Because every structure has different mathematical properties often new solutions arise along with new methods and techniques to solve these equations. One often works to reduce something unknown down to something that is known. Dr. Elfen has been working for the past several years to find new and innovative methods to solve these equations, specifically equations defined on abstract groups. She works in an area in which *a times b* is not the same as *b times a*, and division is not always possible. She has developed new techniques to prove previously proven results and has had recent success in solving functional equations related to stochastic distance measures and quadratic equations. She is currently in collaboration with several mathematicians from the University of Louisville. Their research has recently been published in the journal *Aequationes Mathematicae,* a top journal in the field.

$$\text{Consider the functional equation } f(xy) = f(x)\,f(y),$$
$$\text{where } f : \mathbb{R} \to \mathbb{R} \text{ and } f(x)\,f(y) > 0.$$
$$\text{Then } f(x) = \sqrt{x} \text{ is an example of a solution.}$$
$$\sqrt{xy} = \sqrt{x}\,\sqrt{y}$$

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